Sample

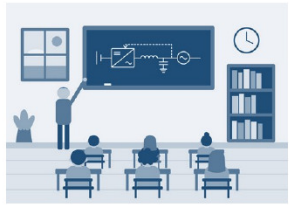
GMF 102: Stability

Short Course Sample Slides

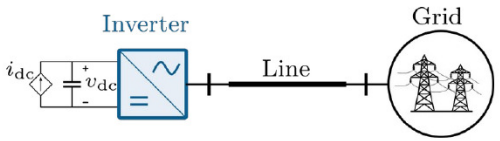
Overview of this short course

Module 1
Introduction to the course

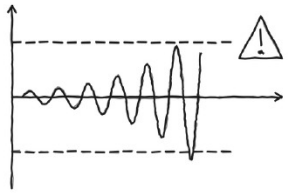
1.1 Welcome to course 102



1.2 Recap from course 101

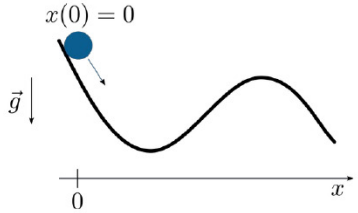


1.3 Why do we need stability analysis?

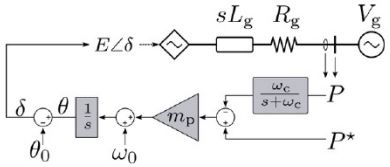


Module 2
Small-signal stability

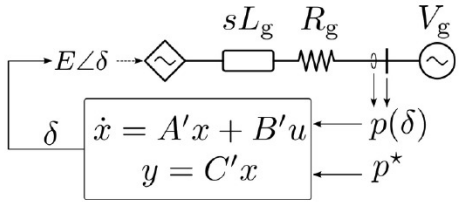
2.1 Intro to small-signal stability of nonlinear dynamical systems



2.2 GFM model abstractions for stability analysis

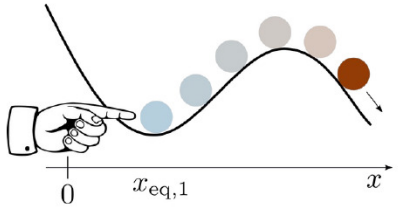


2.3 Small-signal stability of 2nd-order GFM models

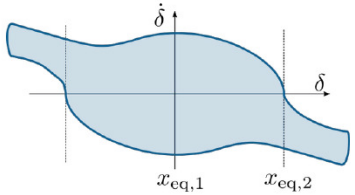


Module 3
Large-signal stability

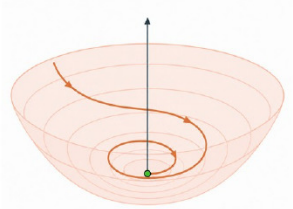
3.1 Intro to large-signal stability of nonlinear dynamical systems



3.2 Large-signal stability concepts applied 2nd-order GFM models

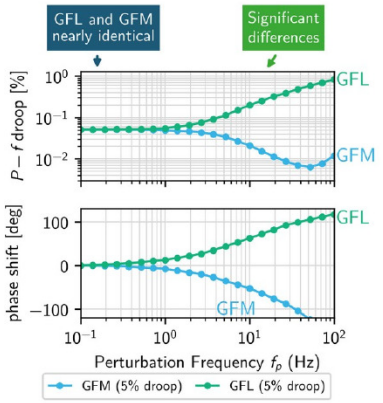


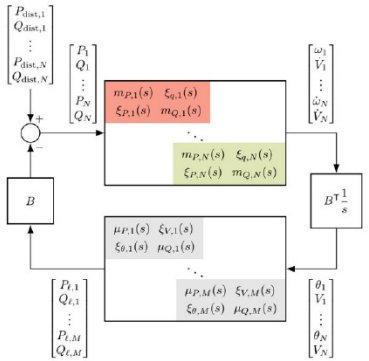
3.3 Estimating the Region of Attraction of 2nd-order GFM models



Module 4
Towards standards

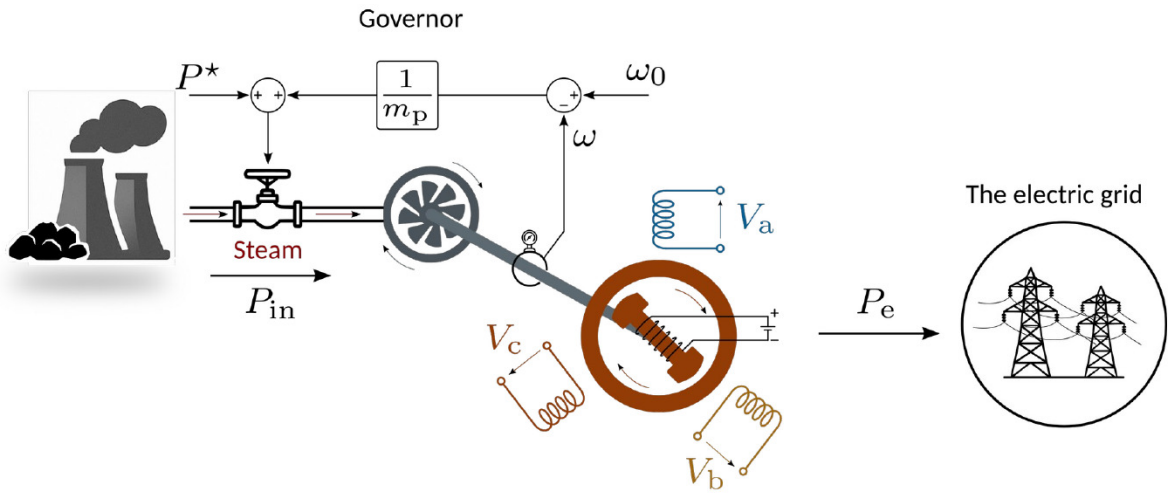
Frequency-domain specifications leading towards verifiable and quantifiable grid codes





Recap: primary control

Primary frequency response in synchronous-generator power plants is facilitated by a **governor**.



$P^\star$ = dispatched active power
ω_0 = nominal / desired frequency of the grid

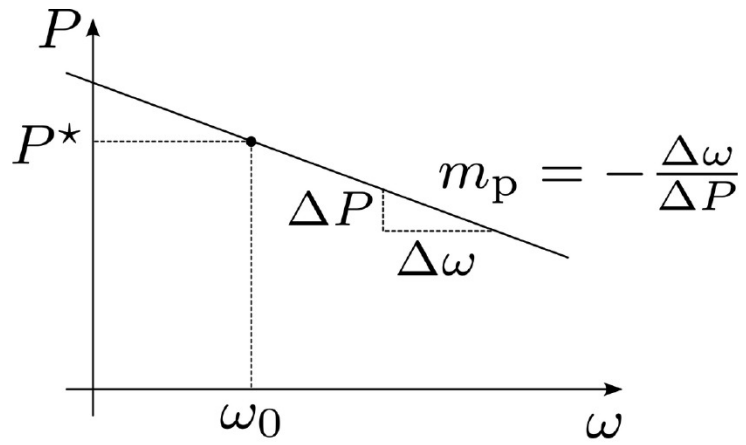
The governor increases the mechanical power input when the rotor frequency drops and visa versa. This stabilizes the frequency during power imbalances.

The governor achieves this through **droop control.**

In governor droop control, active power and rotor frequency are linearly depended:

$$P_\mathrm{in} = P^\star + m_\mathrm{p}^{-1}(\omega_0 - \omega),$$

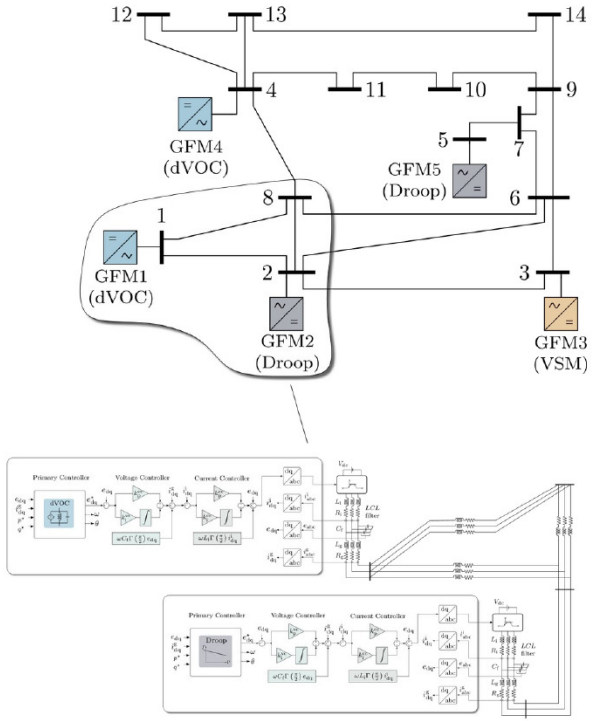
Where m_p = droop gain.



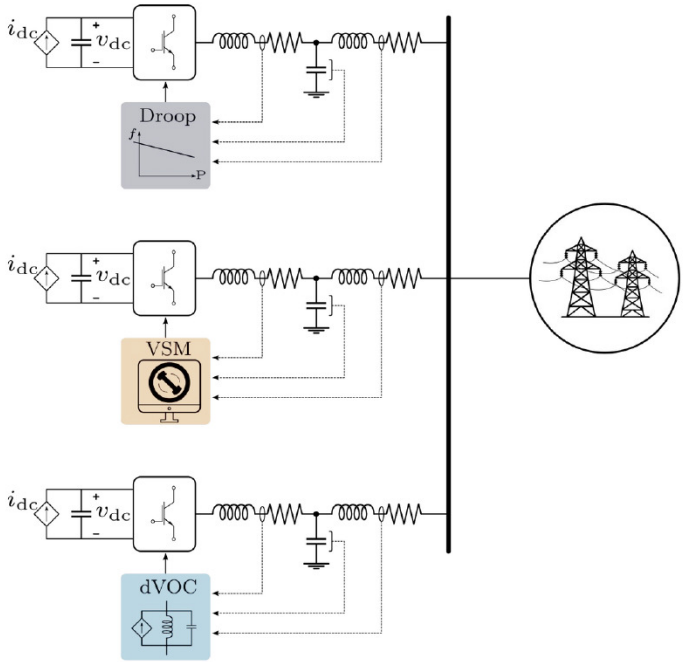
Interconnected GFM inverters

Grid-connected GFM inverters are interconnected devices. To make life easier, we often used simplified setups for analysis and design:

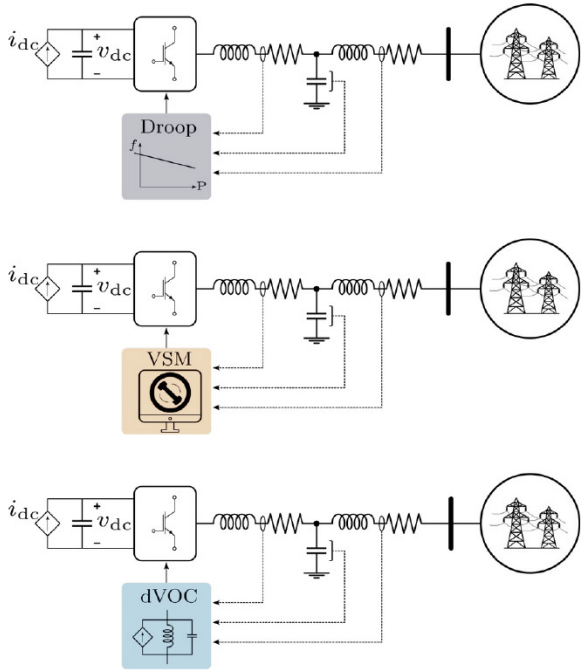
Network-level setup



Inverters are parallel connected

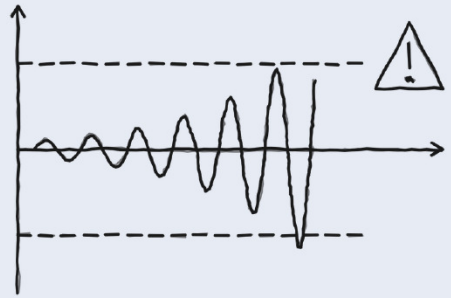


Single-unit-to-infinite bus (SUIB)



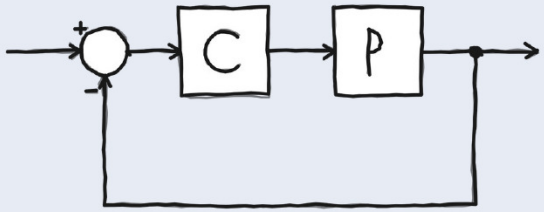
Why do we need to analyze stability?

Safe operation



If the output grows without bounds, it **can lead to violent oscillations** resulting in events such as loss of synchrony.

Design



Stability analysis reveals the limits to which you can stretch the system and design appropriate compensation controls.

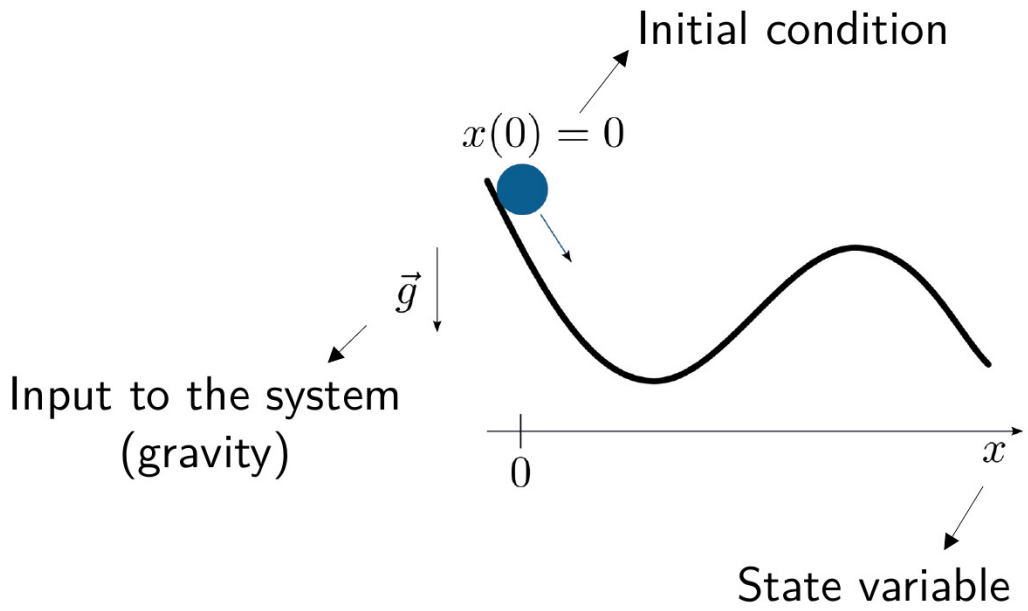
Robustness



Models approximate the real world and need to account for **parameter uncertainty**. Stability margins quantify how much error can be tolerated.

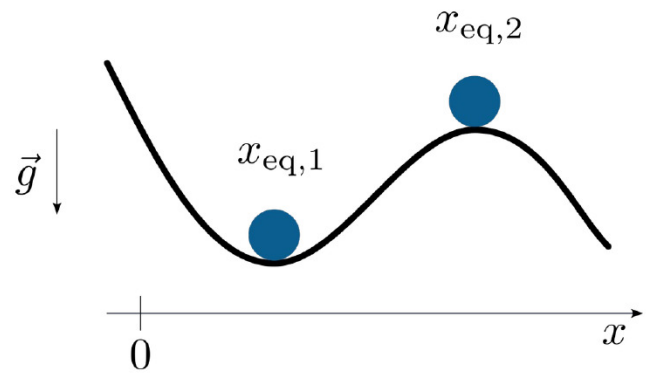
Stability concepts and principles

Consider a nonlinear dynamical system.



In a dynamical system, the **states** represent physical variables that describe the system's condition at any instant and predict its future behavior.

An equilibrium is a point where the dynamics of the system die out. In other words, the state derivatives are zero ($\dot{x} = 0$ identifies $x_{\mathrm{eq},1}$ and $x_{\mathrm{eq},2}$)

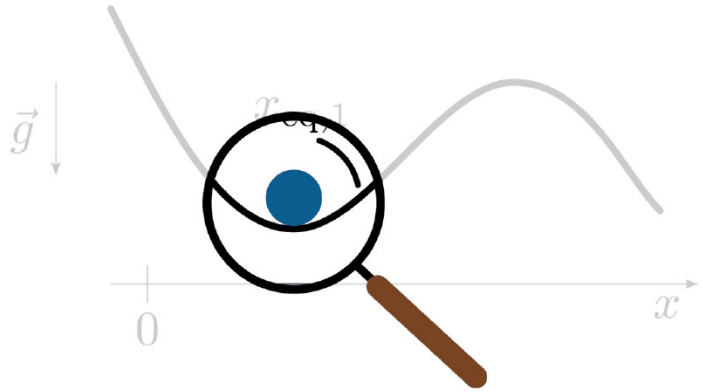


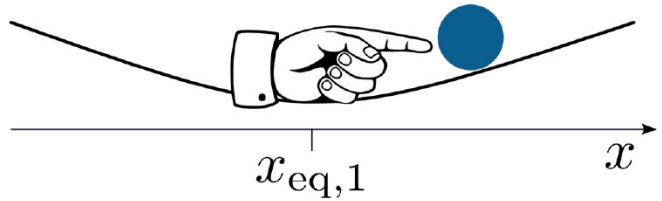
However, different equilibria have different local behavior: **Some are stable, others are unstable**.

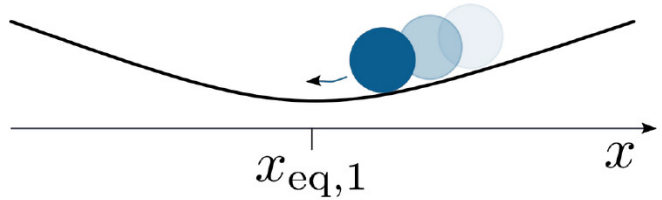
Stability concepts and principles

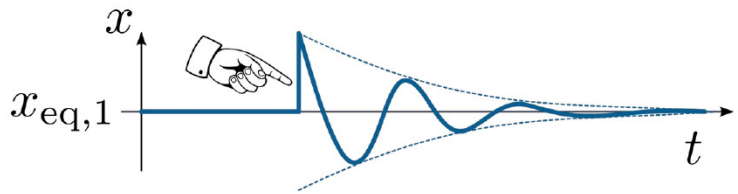
Equilibrium points are the solutions of the algebraic equation $\dot{x} = 0$. How to know if an equilibrium point is stable?

If we look very **closely around the equilibrium**, and we subject the system to a small disturbance – a little push.







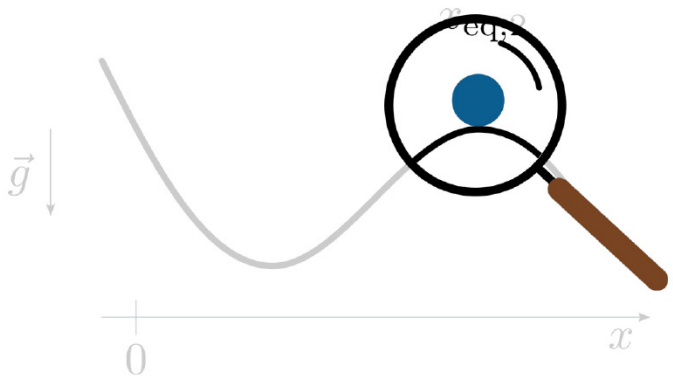


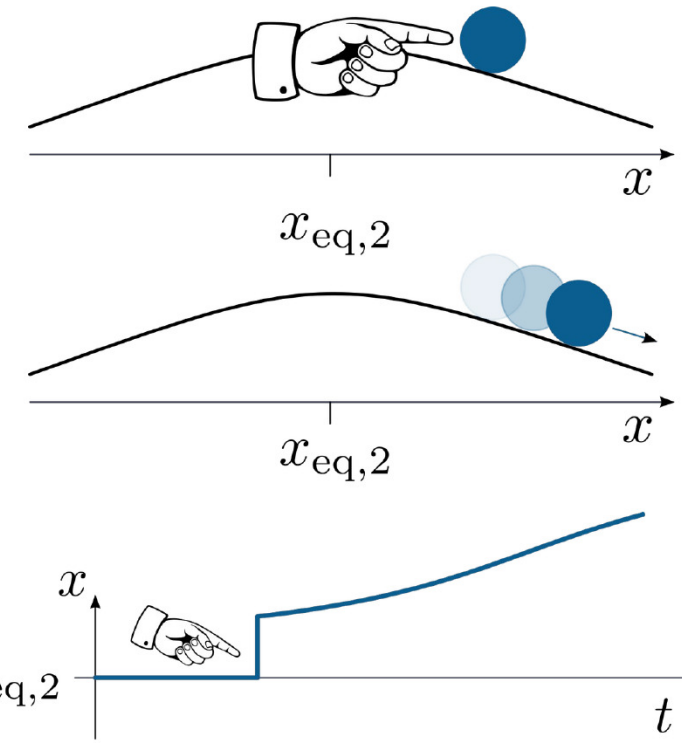
If the **equilibrium is stable**, the system returns to that state after the disturbance

Stability concepts and principles

Equilibrium points are the solutions of the algebraic equation $\dot{x} = 0$. How to know if an equilibrium point is stable?

If we look very **closely around the equilibrium**, and we subject the system to a small disturbance – a little push.

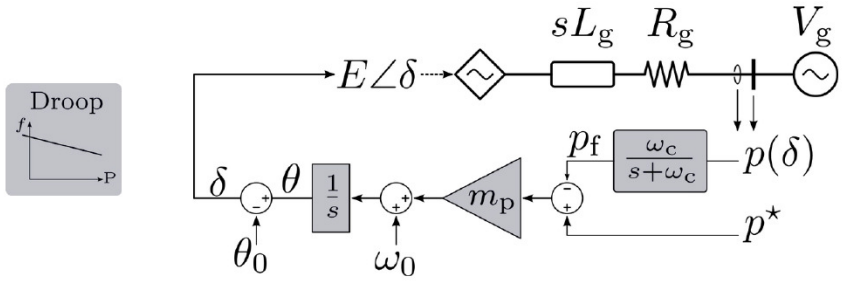




If the **equilibrium is unstable**, the system does not return to that state after the disturbance

Pen-and-paper exercise #1

Derive the **state-space formulation** of a 2nd-order droop GFM model



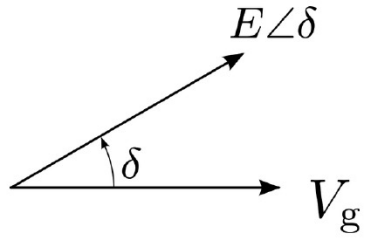
The measured power, p, is filtered, and the frequency is governed by droop:

$$\omega - \omega_0 = m_p(p^\star - p_f)$$
$$\dot{p}_f = \omega_c(p - p_f)$$

where p_f is the filtered output power, ω_c is the cut-off frequency of the low-pass filter, and m_p is the droop gain.

We define $\dot{\delta} = \omega - \omega_0$, such that:

$$\dot{\delta} = m_p(p^\star - p_f) \quad (1)$$
$$\dot{p}_f = \omega_c(p(\delta) - p_f) \quad (2)$$



Rewriting (1) to

$$\dot{\delta} = m_p(p^\star - p_f) \rightarrow \begin{cases} p_f = p^\star - \frac{1}{m_p}\dot{\delta} \\ \dot{p}_f = -\frac{1}{m_p}\ddot{\delta} \end{cases}$$

Substituting the above back into (2) yields

$$-\frac{1}{m_p}\ddot{\delta} = \omega_c(p - p^\star + \frac{1}{m_p}\dot{\delta})$$

Rewrite the above as

$$\ddot{\delta} + \omega_c\dot{\delta} = m_p\omega_c(p^\star - p)$$

Pen-and-paper exercise #1

This is a 2^{nd}-order model with two states. Let the states be δ and $\dot{\delta}$ (the angle and relative frequency of the GFM inverter). We can rewrite the model from before as

$$\ddot{\delta} + \omega_{\text{c}}\dot{\delta} = m_{\text{p}}\omega_{\text{c}}(p^\star - p(\delta)) \quad \rightarrow \quad \begin{bmatrix}\dot{\delta}\\ \ddot{\delta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\omega_{\text{c}}\end{bmatrix}\begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix} + \begin{bmatrix}0\\ m_{\text{p}}\omega_{\text{c}}\end{bmatrix}(p^\star - p)$$

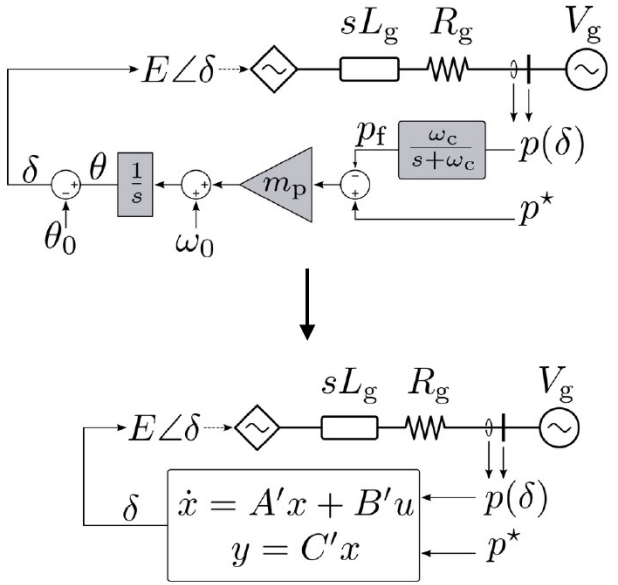
This gives us the final **generalized 2^{nd}-order state-space model** of a GFM inverter:

$$\begin{cases}\dot{x} = A'x + B'u\\ y = C'x\\ u = -\phi(y)\end{cases} \qquad x = \begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix} \quad \text{(the states)}$$

$$y = \delta \quad \text{(the output)}$$

$$A' = \begin{bmatrix}0 & 1\\ 0 & -\omega_{\text{c}}\end{bmatrix}, \quad B' = \begin{bmatrix}0\\ m_{\text{p}}\omega_{\text{c}}\end{bmatrix}, \quad C' = \begin{bmatrix}1 & 0\end{bmatrix}$$

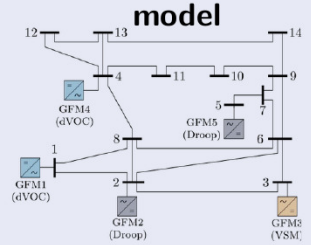
$$\phi(\delta) = p(\delta) - p^\star \quad \text{(the input)}$$



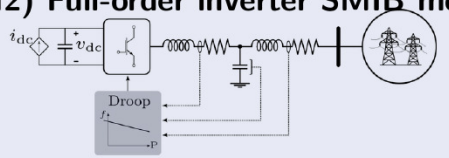
Small-signal stability analysis of GFM inverters

Model Abstractions

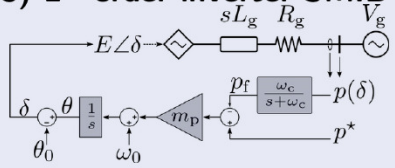
(M1) Full-order network and inverter model



(M2) Full-order inverter SMIB model

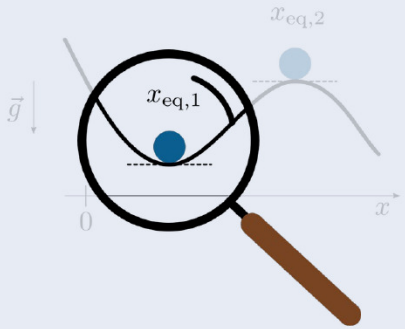


(M3) 2nd-order inverter SMIB model



Linearize nonlinear system around equilibria and find the **eigenvalues**.

$$\Delta\dot{x} = A\Delta x + B\Delta u$$



Small-signal stability of (M1) is **not covered in this course**, but follows the same procedures

Small-signal stability analysis of (M2) is demonstrated through a **MATLAB exercise**

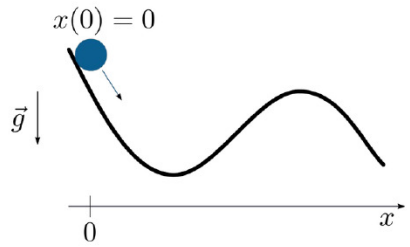
Small-signal stability analysis of (M3) is demonstrated through a **pen-and-paper derivations**

The limitations of small-signal stability analysis

So far, we…

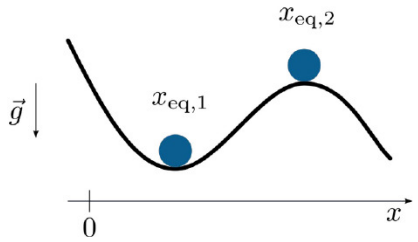
Considered a nonlinear dynamic system

$$\dot{x} = f(x, u)$$



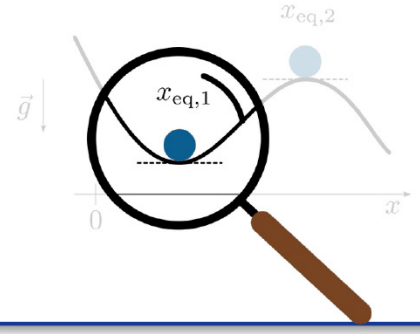
Found equilibrium points

$$\dot{x} = 0 \rightarrow f(x_{\text{eq}}, u_{\text{eq}}) = 0$$

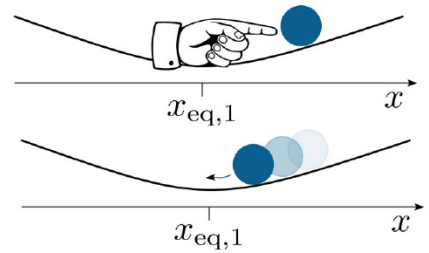


Linearized around the equilibrium points

$$\Delta\dot{x} = A\Delta x + B\Delta u$$



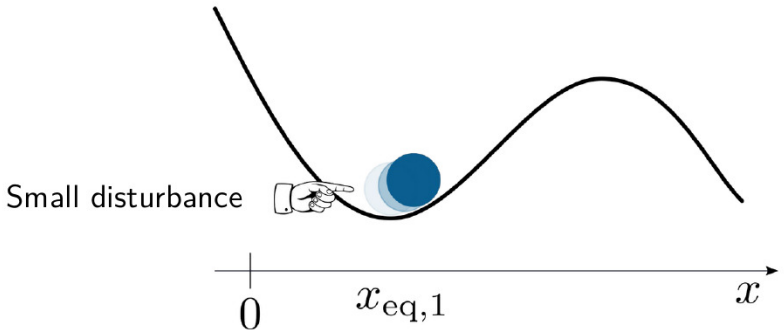
For small perturbations, we compute if equilibrium point is stable (eigenvalues)



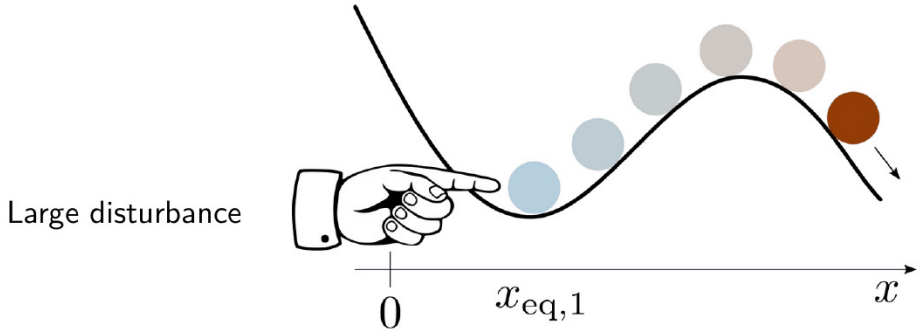
Linearizing around the equilibrium points is justified, because we only consider **small perturbations.**

What if the system is subjected to large perturbations?
Then we can no longer justify to use a linear approximation

The limitations of small-signal stability analysis



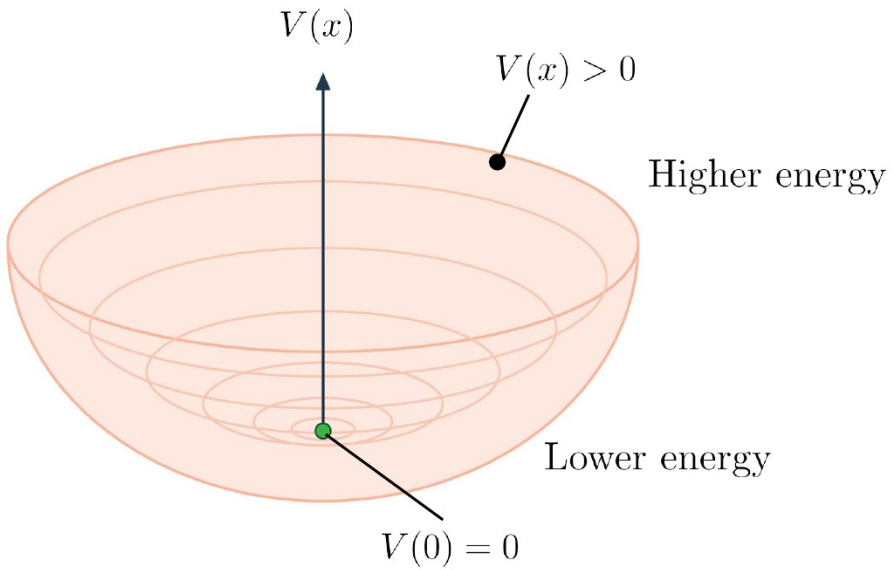
Small-signal stability analysis:
If we look *closely around the equilibrium*, is the equilibrium point stable?



Large-signal stability analysis:
How far can the system move away from the stable equilibrium before it *cannot return to its stable equilibrium*?

Notion of Energy

Notion of energy in mathematical terms:



We denote $V(x)$ as the energy captured by the states.

At the stable equilibrium point, the energy is at its lowest. We say that $V(0) = 0$.

If the energy captured by the states decreases, we know that the system is moving towards equilibrium: $\dot{V}(x) < 0$.